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ABSTRACT

The learning difficulties that students experience with fractions begin immediately when they are shown fraction symbols with one numeral written above the other and told that the "top number" is called the numerator and the "bottom number" is called the denominator. This introduction to fractions will usually include a few visual diagrams to help students learn about these strange new number symbols, and soon thereafter the students are told the rules for operations on them. The purpose of this paper is to show how fraction concepts and operations can be taught intuitively with manipulative materials before fraction symbols and rules are introduced. This is demonstrated with part-to-whole models such as "Fraction Bars." The concepts are built progressively including the part-to-whole concept, equality, addition, subtraction, multiplication and division. In each case, methods and examples are given. It is possible to teach fraction concepts and provide readiness for fraction operations before introducing fraction symbols or rules. Creating and playing games is an effective method of motivating students to become actively involved in the learning process. This paper recommends a lengthy readiness period for fractions and contains specific suggestions for activities during this period. (CW)

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FRACTIONS - CONCEPTS BEFORE SYMBOLS

"Symbols scare people. Instead of presenting mathematics as rigorously as possible, present it as intuitively as possible. In place of abstractions, we must, as far as possible present concrete materials".

Morris Kline

Nowhere in the school curriculum is Morris Kline's advice more applicable then in the teaching of fractions. National and statewide assessment results and related research indicate that many students from middle school to high school have little conceptual understanding of fractions. The National Assessment of Educational Progress (Carpenter, et al. 1981) obtained the following responses from 13-year-olds and 17-year-olds to this test question:

> Estimate the answer to 12/13 + 7/8. You will not have time to solve this problem using paper and pencil.

Responses	Percent Responding Age 13	Age 17
° 1	7	8
• 2	24	37
° 19	28	21
° 21	27	15
° I don't know	14	16

Think for a moment about how the answers of 19 and 21 were obtained. Such answers indicate that students do not understand the concepts underlying computation and that if they do not know a rule or algorithm to solve a problem, they cannot fall back on intuitive models.

The learning difficulties that students experience with fractions begin immediately when they are shown fraction symbols with one numeral written

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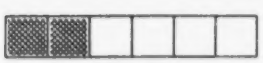
above the other, 3/4, 1/6, 2/3, etc., and told that the "top number" is called the numerator and the "bottom number" is called the denominator. This introduction to fractions will usually include a few visual diagrams to help the students learn about these strange new number symbols, and soon thereafter the students are told the rules for obtaining equal fractions, adding fractions, etc.

The purpose of this article is to show how fraction concepts and operations can be taught intuitively with manipulative materials before fraction symbols and rules are introduced. While this will be accomplished with the Fraction Bars model, other part-to-whole models, such as Fraction Tiles, Pattern Blocks, or Fraction Pieces may be used as well (McBride and Lamb 1986). The Fraction Bars model has color-coded bars with 2 parts, 3 parts, 4 parts, 6 parts, and 12 parts. With all the different possible shaded amounts there are 32 different bars. If these bars are not available, they can be made from colored posterboard, or the students can make their own bars by paper folding strips of paper and shading the parts (Scott 1981).

Part-to-Whole Concept Each of the following bars can be described by the total number of parts and the number of shaded parts.



A red bar with 2 shaded parts, or a bar with 2 shaded parts out of 6.

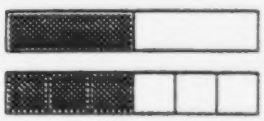
A blue bar with 3 shaded parts, or a bar with 3 shaded parts out of 4.

A yellow bar with 1 shaded part, or a bar with 1 shaded part out of 3.

Fig. 1.

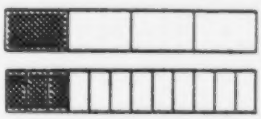
Bars can be held up by the students, or transparent bars placed on the overhead, and verbally described by the class. Or, the teacher can describe a bar and have the students find it. Students can be asked to find specific types of bars, such as a bar with all its parts shaded (a whole bar), or a bar with no shading (a zero bar).

Equality The bars can be compared for equal shaded amounts by lining them up vertically.



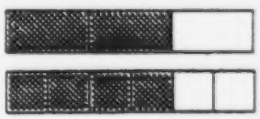
1 part out of 2 has the same amount of shading as 3 parts out of 6.

(a)



1 part out of 4 has the same amount of shading as 3 parts out of 12.

(b)

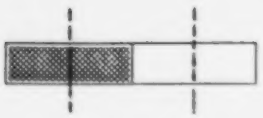


2 parts out of 3 has the same amount of shading as 4 parts out of 6.

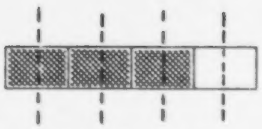
(c)

Fig. 2.

One student can select a bar and the other students can try to find a different colored bar with the same shaded amount. Sometimes there will be several different colored bars with the same shaded amount: 1 part out of 3; 2 parts out of 6; and 4 parts out of 12 all have the same shaded amount. The students can sort the bars into piles according to their shaded amounts. After the students are familiar with the bars, the teacher can show them a bar with 1 part shaded out of 2, and ask them to visualize a different bar with the same amount of shading. One way to get this new bar is to imagine drawing vertical lines to divide each part of the bar into 2 equal parts (figure 3). If transparent bars are available this can be demonstrated on the overhead.



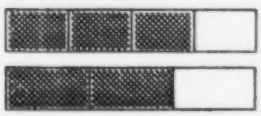
1 part out of 2 is equal to
2 parts out of 4.
(a)



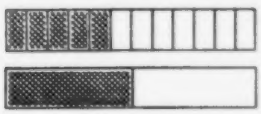
3 parts out of 4 is equal to
6 parts out of 8.
(b)

Fig. 3

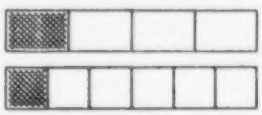
Inequality The bars can be compared for unequal shaded amounts by lining them up vertically.



3 parts out of 4 is greater than 2 parts out of 3.
(a)



5 parts out of 12 is less than 1 part out of 2.
(b)



1 part out of 4 is greater than 1 part out of 6.
(c)

Fig. 4.

Each student can select a bar and compare it to that of another student's to determine the greater shaded amount. This activity can be turned into a game with the student who has the greater shaded amount winning both bars. The teacher can show the students a bar with 1 part out of 2 shaded and ask them to describe a bar which is less then half shaded. This can be followed by a discussion of how to determine when a bar will be less than half shaded, if the number of parts and the number of shaded parts are known. Arranging the bars as shown here so that the vertical lines are lined up, provides opportunities for discovering both equality and inequality relationships. A discussion of decreasing shaded amounts will help

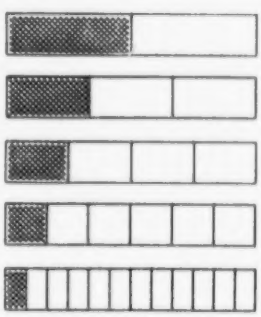
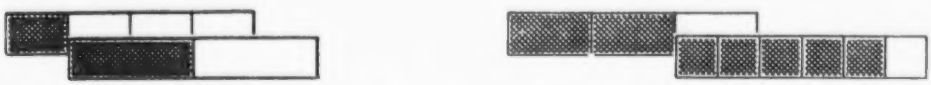


Fig. 5.

students to see that the greater the number of equal parts in a bar, the smaller the parts: 1 part out of 4 is less than 1 part out of 3; 1 part out of 12 is less than 1 part out of 4; etc. Students should be encouraged to offer reasons for such inequalities. For example, if a candy bar is shared equally among four people the parts are smaller than if it is shared among three people. The game FRIO (Drizigacher, 1966) if played with the bars, will involve increasing and decreasing shaded amounts.

Addition Placing the shaded parts of two bars end to end shows that the total shaded amount of the bars.



1 part out of 4 and 1 part out of 2 have a total shaded amount of 3 parts out of 4.

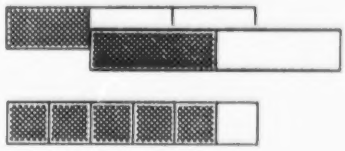
(a)

2 parts out of 3 and 5 parts out of 6 have a total shaded amount of 1 whole bar and 3 parts out of 6.

(b)

Fig. 6.

It is important for concept formation to compare the total shaded amount of two bars to a whole bar. In figure 7 the total shaded amount is more than half a bar and less than a whole bar. For such bars, where the vertical lines do not line up, the total shaded amount can always be found by using a third bar, as shown.

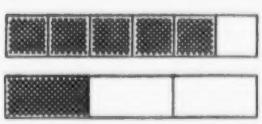


1 part out of 3 and 1 part out of 2 have a total shaded amount of 5 parts out of 6

Fig. 7.

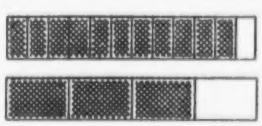
For an activity, each student can select two bars and determine the total shaded amount. This also can be done with transparent bars on the overhead and can lead to discussions of different ways to approximate or determine the total shaded amount of the two bars. The students can compare the total shaded amounts of their bars to determine who has the greatest. This activity can be played as a game. In another version of this game, each student selects bars, one at a time, and tries to get a total shaded amount which is as close as possible to one whole bar, without going over. The bars can provide strong visual support for approximations. Students should be encouraged to visualize two bars and approximate the total shaded amount as being less than a whole bar, more than a whole bar, or close to two whole bars. For example, the total shaded amount of a bar with 7 parts shaded out of 8 and a bar with 12 parts shaded out of 13 is close to two whole bars.

Subtraction The bars can be compared for different shaded amounts by lining them up vertically.



The difference in shaded amounts between 5 parts out of 6 and 1 part out of 3 is 3 parts out of 6.

(a)



The difference in shaded amounts between 11 parts out of 12 and 3 parts out of 4 is 2 parts out of 12.

(b)

Fig. 8.

Placing the bars vertically makes it natural to view differences in terms of missing addends. That is, what shaded amount must be added to one bar to equal the shaded amount of the other. For example, in figure 8(a) 3 parts out

of 6 must be added to 1 part out of 3. Whenever the vertical lines of the bars do not line up, a third bar can be used to determine the difference in shaded amounts. In figure 9 the difference in shaded amounts between the top two bars is 1 part out of 6. Some students would not need the third bar in this example because they would know that half of 1 part out of 3 is 1 part out of 6. For an activity each student can select two bars and determine the difference in the shaded amounts. This can lead to a discussion of ways to determine some of these differences. The students should be encouraged to determine differences in special cases by visualizing two bars. For example, the teacher might ask the students to visualize a bar with 6 parts shaded out of 10 and 1 part shaded out of 2 and to determine the difference in their shaded amounts.

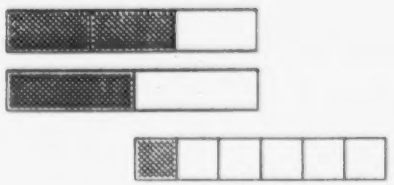
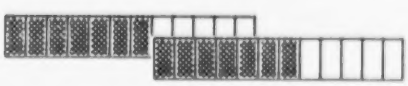


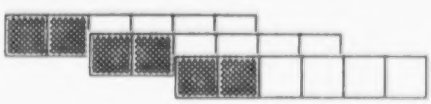
Fig. 9.

Multiplication The total shaded amount of several copies of the same bar can be determined by placing their shaded amounts end to end.



The total shaded amount of two bars, each with 7 parts out of 12, is a whole bar and 2 parts out of 12.

(a)



The total shaded amount of three bars, each with 2 parts out of 6, is 6 parts out of 6 or 1 whole bar.

(b)

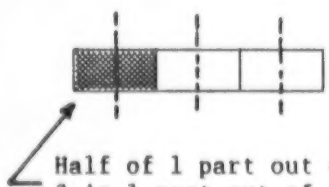
Fig. 10.

The students can determine the total shaded amount in these "repeated addition" type problems by first computing the total number of shaded parts

and then converting to whole numbers and parts of a bar. Consider determining the total shaded amount of four bars, if each bar has 2 parts shaded out of 3. Since there is a total of 4 x 2 = 8 shaded parts, and there are 3 parts in one whole bar, the total shaded amount is 2 whole bars and 2 parts out of 3.

Another type of multiplication problem can be posed by asking questions similar to the following:

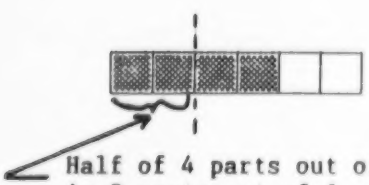
Mary has a bar with 1 part out of 3 shaded.
She gives half of the shaded part away.
What part of a whole bar has she given away?



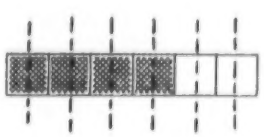
To answer this question the shaded part of the bar in figure 11 has been split into two equal parts. One of these parts is 1 part out of 6.

Fig. 11.

Sometimes it helps to split each of the remaining parts of the bar into two equal parts, as in figure 11, to show the number of new parts in the whole bar. In some cases students will be able to see more than one way of finding half of the shaded amount of a bar, as illustrated in figure 12.



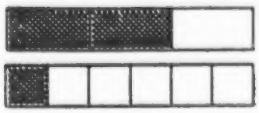
(a)



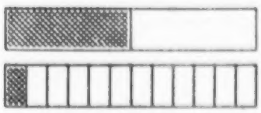
(b)

Fig. 12.

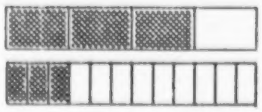
Division The number of times greater the shaded amount of one bar is than the shaded amount of another can be determined by lining the bars up vertically.



1 part out of 6 "goes into" 2 parts out of 3 four times.
(a)



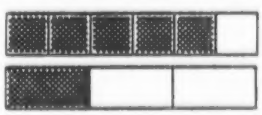
1 part out of 12 "goes into" 1 part out of 2 six times.
(b)



3 parts out of 12 "goes into" 3 parts out of 4 three times.
(c)

Fig. 13.

To help the students become acquainted with the concept of comparing two bars, it is helpful to select bars for which the shaded amount of one is a whole number of times greater than the shaded amount of the other, as illustrated in figure 13. However, some students may be ready for situations in which the quotient is not a whole number, as shown in figure 14. In special cases students will be able to answer questions by visualizing bars and using their knowledge of the bars. For example, How many times will 1 part out of 6 "go into" 1 whole bar? or, How many times will 1 part out of 4 "go into" 1 whole bar? The bars help to show why it is possible to divide one small amount by another and get a number greater than 1. This would be valuable to discuss with the students through examples. For instance, 1 part out of 4 "goes into" 1 part out of 2 two times; 1 part out of 6 "goes into 1 part out of 2 three times; and 1 part out of 12 "goes into 1 part out of 2 six times; etc. Through such examples students can be led to see that as one amount remains fixed and the divisor gets smaller, the quotient gets bigger.



1 part out of 3 "goes into" 5 parts out of 6 two and a half times.

Fig. 14.

Closing Remarks It is possible to teach fraction concepts and provide readiness for fraction operations before introducing fraction symbols or

fraction rules. To supplement teacher directed activities, creating and playing games is an effective method of motivating students to become actively involved in the learning process (Bennett and Schiot 1985). The concrete-visual period with manipulatives should occur for an extended time. Without an adequate presymbol period in which students can become familiar with fraction concepts, many students will rely solely on rote procedures when they start using fractions. The Minnesota State Department of Education and the Minnesota Council of Teachers of Mathematics (1976) proposes using manipulatives to teach fractions throughout the elementary school years and a more gradual transition from concepts of fractions to computation. After a lengthy readiness period students will appreciate the convenience of fractions for recording fraction facts which they know and understand. Even when students are practicing fraction skills and problem solving with fractions, they should be continually encouraged to think about the meaning of symbols and operations in terms of manipulative materials and models. The National Assessment of Educational Progress (Carpenter, et al. 1981) found that 33% of the 13-year-olds thought that 1/2 + 1/3 was equal to 2/5. If students were to think of this sum as the total shaded amount of the bars for 1/2 and 1/3, and remember that the bar for 2/5 is less than half shaded, they would know that 2/5 is not a reasonable answer. In discussing the results of the National Assessment of Educational Progress, Tom Post (1981) states, "It is apparent that more effort needs to be expended helping students to internalize the concept of fraction." In his list of common characteristics of successful attempts to teach fractions he includes: incorporating various types of manipulative materials in the instructional process; and placing a premium on the development of understanding and meaning prior to formal work with symbols and operations.

"Research has shown that sensible use of concrete materials is effective in teaching rational numbers" (Driscoll 1984). There is mounting evidence of the need for teaching understanding of fraction concepts, and of the ineffectiveness of reteaching fractions in grade after grade without substantial use of manipulative materials and models. Perhaps it is time to seriously consider teaching fraction concepts before introducing fraction symbols.

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